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Two-pion production processes, chiral symmetry and NN interaction in the medium

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Abstract. We study the two-pion propagator in the nuclear medium. This quantity appears in the $\pi\pi$ T-matrix and we show that it also enters the QCD scalar susceptibility. The medium effects on this propagator are due to the influence of the individual nucleon response to a scalar field through their pion clouds. This response is appreciably increased by the nuclear environment. It produces an important convergence effect between the scalar and pseudoscalar susceptibilities, reflecting the reshaping of the scalar strength observed in 2π production experiments. While a large modifications of the σ propagator follows, due to its coupling to two pion states, we show that the NN potential remains instead unaffected.

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1 Introduction

The two-pion production experiments on nuclei [1–4] have revealed a striking accumulation of scalar strength for the 2π invariant mass near threshold, restricted to the isoscalar channel for the two pions. Schuck *et al.* [5] and Chanfray *et al.* [6] have predicted such effects on the basis of the influence of the modification of the pion dispersion relation in the medium on the scalar strength distribution. The pion line is replaced by a pion branch, a collective mixture of pions and Δ-hole states, which lies at lower energies. It leads to a strength concentration near the 2π threshold for the sigma meson which decays in two pions. Other interpretations have been given in refs. [7–9]. Focusing on the interpretation of refs. [5] and [6], one of our aims is to establish the existence of a link, albeit not a straightforward one, between the softening of the scalar strength and chiral symmetry restoration. The link goes as follows: the enhancement of the $\pi\pi$ T-matrix near threshold arises from the nuclear modification of the two-pion propagator. This also affects the QCD scalar susceptibility, which will be one of our topics.

Beside 2π states, the lowest excitations modes of the vacuum which govern the scalar susceptibility can also imply a genuine scalar-isoscalar meson, the so-called "sigma" meson. These aspects can be incorporated in the linear sigma model, with a σ chiral partner of the pion strongly coupled to 2π states. We will discuss how the propagation of the sigma meson is affected by the modification of the two-pion propagator.

In this model the sigma propagator governs the scalar susceptibility. Our results thus have an implication for the in-medium modification of this quantity. The nuclear-matter susceptibility has previously been discussed in particular in refs. [10,11]. The results that we will derive here for the susceptibility are in part implicitly contained in ref. [11]. In order to display this link one has to isolate in the nucleonic sigma commutator its pionic component, as was indeed done in ref. [11]. But this was done for another purpose (to single out the dominant pion cloud contribution to the nucleonic scalar susceptibility) and the link with the 2π production experiments did not naturally emerge. Here we will show that the two-pion propagator is a component of the sigma propagator which governs the scalar susceptibility. This naturally provides the connection to the $\pi\pi$ T-matrix. Moreover, we evaluate the medium effects on the two-pion propagator to all orders in density while in previous papers only the linear term in density was included.

The other point that we will elucidate is the connection with traditional aspects of the nuclear binding since the strong in-medium reshaping of the scalar strength has *a priori* consequences for this problem. Our discussion will be based on the distinction, previously emphasized [12], between the sigma, chiral partner of the pion, and the scalar meson exchange of nuclear physics. We will show

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that the other component of the sigma propagator is related to the propagation of the chiral invariant s field which enters the NN interaction. The s exchange part of this interaction is thus totally disconnected from the medium effects on the two-pion propagator, which has implications for the in-medium NN interaction.

2 T-matrix, two-pion propagator and scalar susceptibility in the medium

In the 2π production experiments the medium effects are governed by the in-medium modifications of the T-matrix for the $\pi\pi$ scattering. In the following we will study this quantity in a chiral model, the linear sigma one. In fact our results on the medium effects do not depend on the scalar meson mass but essentially on the two-pion propagator that could be obtained directly in the non-linear sigma model. It is interesting, however, to keep a sigma meson with a finite mass in order to establish link with the binding properties of nuclei.

In this model the coupling of the σ to two-pion states is a simple 3-point vertex, $\lambda f_\pi \sigma\pi^2$, while the four-pion interaction is $\lambda\pi^4/4$. The coupling constant λ is related to observables according to $\lambda = (m_\sigma^2 - m_\pi^2)/2f_\pi^2$. The corresponding s-channel contribution, V_s, to the $\pi\pi$ potential at a given invariant squared mass s is the sum of a contact term and a sigma exchange one. The same structure also holds for the t and u channels. We use an approximation suggested by the authors of ref. [13], who keep only the s-channel term, dropping the t and u channel contributions which enter with a smaller weight in the isoscalar channel. These authors have shown that, within a symmetry-conserving $1/N$ expansion (here $N = 4$) fulfilling Ward identities, this is a legitimate approximation. Within this simplified framework they were able to reproduce the $\pi\pi$ phase shifts and scattering length [13]. Then the scalar-isoscalar potential reads

$$V = 6\,\lambda + 12(\lambda f_\pi)^2 \frac{1}{s - m_\sigma^2} = 6\,\lambda \frac{E^2 - m_\pi^2}{E^2 - m_\sigma^2}\,, \tag{1}$$

being $E = \sqrt{s}$ the total energy of the pion pair in the CM frame.

The Lippmann-Schwinger equation with such a separable potential gives for the unitarized scalar-isoscalar T-matrix

$$T(E) = \frac{6\lambda(E^2 - m_\pi^2)}{E^2 - m_\sigma^2 - 3\lambda(E^2 - m_\pi^2)G(E)}\,, \tag{2}$$

which in the limit of large sigma mass reduces to (assuming zero three-momentum for the pion pair)

$$T(E) = -\frac{3(E^2 - m_\pi^2)/f_\pi^2}{1 + 3(E^2 - m_\pi^2)/(2f_\pi^2)G(E)}\,. \tag{3}$$

Here $G(E)$ is the two-pion propagator for zero three-momentum of the pion pair, linked to the single-pion propagator D_π by

$$G(E) = \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{i dq_0}{2\pi} D_\pi(\mathbf{q}, q_0) D_\pi(-\mathbf{q}, E - q_0). \tag{4}$$

The expression of $T(E)$ given in eq. (2) holds either in the vacuum or in the medium. In the last case the pion propagators which enter G are dressed by particle-hole bubbles, where the particle is either a nucleon or a Delta. This is responsible for the modification of the T-matrix in the medium and, in the interpretation of refs. [5,6,14], explains the features of the data.

We now come to the link with the chiral symmetry. The two-pion propagator is the correlator of a scalar quantity, the squared pion field. On the other hand, the order parameter for the spontaneous breaking of the chiral symmetry is another scalar, the quark condensate, that we denote $\langle\bar{q}q(\rho)\rangle$ in the nuclear medium, whereas $\Delta\langle\bar{q}q(\rho)\rangle$ is the variation with respect to the vacuum. The QCD scalar susceptibility, defined as the derivative of the quark condensate with respect to the quark mass, is the correlator of the fluctuation of the quark scalar density. We define the nuclear value χ_S^A as the difference with the vacuum quantity

$$\chi_S^A(\rho) = \frac{d}{dm_q} \Delta\langle\bar{q}q(\rho)\rangle. \tag{5}$$

One can expect a link between the two scalar correlators: the QCD susceptibility and the two-pion propagator. Indeed, a major contributor to chiral symmetry restoration is the nuclear pion cloud. In the non-linear sigma model this is the only agent for restoration. Each pion of the cloud contributing to the restoration by an amount proportional to the sigma commutator of the pion, $\Sigma_\pi = m_\pi/2$, the pionic participation to the restoration is expressed in terms of the pion density in the cloud, $m_\pi\langle\Phi^2\rangle$, according to [15]:

$$\Delta\langle\bar{q}q(\rho)\rangle = -\langle\bar{q}q\rangle_{vac} \frac{\langle\Phi^2\rangle}{2f_\pi^2}. \tag{6}$$

The quantity $\langle\Phi^2\rangle$ is related to the pion propagator $D_\pi(q)$ by

$$\begin{aligned}\langle\Phi^2\rangle &= 3\int \frac{i d^4q}{(2\pi)^4}[D_\pi(q) - D_{0\pi}(q)] \\ &= 3\int \frac{i d^4q}{(2\pi)^4}\left[\frac{1}{q^2 - m_\pi^2 - S_\pi(q)} - \frac{1}{q^2 - m_\pi^2}\right].\end{aligned} \tag{7}$$

In the expression above the vacuum value of the pion propagator $D_{0\pi}(q)$ is subtracted out in order to retain only medium effects and $D_\pi(q)$ is related to the pion self-energy $S_\pi(q)$, which includes an s-wave, S_s, and a p-wave part, S_p. The second piece arises from the p-wave excitations of particle-hole, yielding a three-momentum–dependent coupling of the pion. It does not depend explicitly on the pion mass, while the s-wave one does.

For the evaluation of the susceptibility the derivative with respect to the quark mass is replaced as usual by the one with respect to the pion mass squared. We ignore the derivative of the s-wave potential which leads to small corrective terms. With this approximation we obtain a

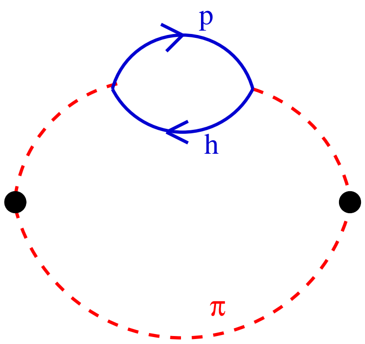

Fig. 1. Influence of the nucleonic pion cloud on the two-pion propagator. For the susceptibility the quark fluctuation is attached at each point.

simple expression for the susceptibility

$$\begin{aligned}\chi_S^A(\rho) &= \frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\frac{d}{dm_\pi^2}\langle\Phi^2\rangle\\ &= 3\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\int\frac{\mathrm{i}d^4q}{(2\pi)^4}\left[D_\pi^2(q)-D_{0\pi}^2(q)\right]\\ &= 3\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\Delta G(0),\end{aligned}\tag{8}$$

which is proportional to $\Delta G(0)$, the in-medium modification, at $E=0$, of the two-pion propagator, the quantity which governs the modification of the $\pi\pi$ T-matrix.

In order to illustrate the significance of this contribution we first consider a single insertion of a nucleon-hole bubble (namely Π_N^0) into one of the two-pion lines (see fig. 1). The corresponding medium correction reads

$$\chi_S^A(\rho) = 3\frac{2\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\int\frac{\mathrm{i}d^4q}{(2\pi)^4}D_{0\pi}(-q)D_{0\pi}^2(q)\mathbf{q}^2\,\Pi_N^0(q).\tag{9}$$

The physical interpretation follows from fig. 1: this medium correction introduces the effect of the individual nucleonic susceptibility from their pion clouds. Note that the Pauli-blocking effect is implicitly contained through the quantity Π_N^0. Ignoring it, the contribution to the scalar susceptibility is $\rho_s\chi_S^N(\pi)$, where ρ_s is the nucleon scalar density and we have denoted $\chi_S^N(\pi)$ the free-nucleon susceptibility from its pion cloud. This quantity was discussed by Chanfray *et al.* [11] who showed that it dominates the nucleon response and who evaluated it in the static approximation: $\chi_S^N=-4.10^{-2}\,\mathrm{MeV}^{-1}$.

In the description above we have considered in the two-pion propagator the dressing of a single-pion line by only one bubble. If instead we introduce in the two-pion propagator the full RPA pion propagator in the ring approximation, the contribution to the nuclear susceptibility can be written as $\rho_s\tilde{\chi}_S^N$, with an in-medium–modified, density-dependent nucleonic scalar susceptibility $\tilde{\chi}_S^N$. The relation between ΔG and $\tilde{\chi}_S^N$ is

$$3\Delta G(0) = \frac{\rho_s\tilde{\chi}_S^N f_\pi^4}{\langle\bar{q}q\rangle_{vac}^2}\,.\tag{10}$$

At ρ_0 it happens that $3\Delta G(0)\simeq\tilde{\chi}_S^N$, if $\tilde{\chi}_S^N$ is expressed in MeV^{-1}. For its evaluation we proceed as follows. First, the complete bare polarization propagator Π^0 is the sum of the nucleon-hole polarization propagator $\Pi_N^0(q)$ and of the Delta-hole one, namely

$$\begin{aligned}\Pi_\Delta^0(\mathbf{q},\omega)=&\frac{4}{9}\left(\frac{g_A}{f_\pi}\right)^2\left(\frac{g_{\pi N\Delta}}{g_{\pi NN}}\right)^2F^2(\mathbf{q},\omega)\int\frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\theta(k_F-k)\\ &\times\left(\frac{1}{\omega+\epsilon_{\mathbf{k}}-\epsilon_{\Delta,\mathbf{k}+\mathbf{q}}+i\Gamma_\Delta(\mathbf{k}+\mathbf{q},\omega)}-\frac{1}{\omega+\epsilon_{\Delta,\mathbf{k}-\mathbf{q}}-\epsilon_{\mathbf{k}}}\right),\end{aligned}\tag{11}$$

where $g_{\pi NN}$ ($g_{\pi N\Delta}$) is the πNN ($\pi N\Delta$) coupling constant, $F(\mathbf{q},\omega)$ is the form factor at the πNN or $\pi N\Delta$ vertex and Γ_Δ is the Delta width (taken following ref. [16]). Moreover, we have defined $\epsilon_{\mathbf{k}}=\mathbf{k}^2/2M_N$ and $\epsilon_{\Delta,\mathbf{k}}=M_\Delta-M_N+(\mathbf{k}^2/2M_\Delta)$.

The (p-wave) pion self-energy is linked to the fully dressed polarization propagator, $\tilde{\Pi}$, solution of the RPA equations in the ring approximation, as follows:

see eq. (12) *on next page*

where g'_{NN}, $g'_{\Delta\Delta}$, $g'_{N\Delta}$ are the Landau-Migdal parameters for the NN channel, for the $\Delta\Delta$ one and for the mixing of NN and ΔN excitations, respectively.

Inserting this expression of the pion self-energy into the two-pion propagator of eq. (8) we obtain the nuclear susceptibility, hence the effective nucleonic one. It is interesting to compare the effective and the free-nucleon susceptibilities. For the last quantity we want to avoid the static approximation of Chanfray *et al.* [11]. We can use the same expression (8) in the dilute limit, $\rho\to 0$, so as to eliminate the influence of the medium on the susceptibility.

We have also introduced another method starting from the general expression of the pion density in terms of the spin-isospin longitudinal response function, R_L, as given in ref. [17],

$$\begin{aligned}\langle\Phi^2\rangle &= \frac{3\rho}{A}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\\ &\times\int_0^\infty\mathrm{d}\omega\left(\frac{1}{2\omega_q^2(\omega+\omega_q)^2}+\frac{1}{2\omega_q^3(\omega+\omega_q)}\right)R_L(\mathbf{q},\omega)\end{aligned}\tag{13}$$

with $\omega_q=\sqrt{\mathbf{q}^2+m_\pi^2}$. The response R_L is

$$\begin{aligned}R_L(\mathbf{q},\omega) &= -\frac{V}{\pi}\,\mathrm{Im}\,\Pi_L(\mathbf{q},\omega)\\ &= -\frac{V}{\pi}\,\mathrm{Im}\left(\mathbf{q}^2\tilde{\Pi}(\mathbf{q},\omega)\frac{\omega^2-\omega_q^2}{\omega^2-\omega_q^2-\mathbf{q}^2\tilde{\Pi}(\mathbf{q},\omega)}\right).\end{aligned}\tag{14}$$

Inserting this expression (13) of $\langle\Phi^2\rangle$ into the quark condensate of eq. (6) and taking the derivative with respect to the pion mass squared gives the nuclear susceptibility. Expression (13) holds in the dense case as well as in the dilute limit, *i.e.*, for an assembly of independent nucleons. In the latter case the expression of the response simplifies

$$S_\pi(\mathbf{q},\omega)=\mathbf{q}^2\tilde{\Pi}(\mathbf{q},\omega)=\mathbf{q}^2\frac{\Pi^0_N(\mathbf{q},\omega)+\Pi^0_\Delta(\mathbf{q},\omega)-(g'_{NN}+g'_{\Delta\Delta}-2g'_{N\Delta})\Pi^0_N(\mathbf{q},\omega)\Pi^0_\Delta(\mathbf{q},\omega)}{[1-g'_{NN}\Pi^0_N(\mathbf{q},\omega)][1-g'_{\Delta\Delta}\Pi^0_\Delta(\mathbf{q},\omega)]-g'^2_{N\Delta}\Pi^0_N(\mathbf{q},\omega)\Pi^0_\Delta(\mathbf{q},\omega)}\,, \qquad (12)$$

Table 1. The effective nucleonic scalar susceptibility in unit of $10^{-2}\,\mathrm{MeV}^{-1}$ at normal nuclear-matter density compared to the free-nucleon one (zero-density column) for different values of Λ and for two values of $(g_{\pi N\Delta}/g_{\pi NN})^2$ (corresponding, respectively, to the current nuclear phenomenology and to the constituent quark model) and $g'_{N\Delta}=g'_{\Delta\Delta}\equiv g'_\Delta$. The parameter g'_{NN} is kept at the fixed value $g'_{NN}=0.7$.

Λ MeV	$\rho=0$	$g'_\Delta=0.4$	$g'_\Delta=0.5$	$\rho=0$	$g'_\Delta=0.4$	$g'_\Delta=0.5$
5 m_π	−5.5	−7.3	−6.5	−5.0	−5.9	−5.4
800	−6.0	−8.8	−7.7	−5.4	−7.0	−6.4
900	−6.4	−10.0	−8.7	−5.8	−7.8	−7.0
1000	−6.7	−11.3	−9.6	−6.1	−8.6	−7.3
	$(\frac{g_{\pi N\Delta}}{g_{\pi NN}})^2=3.8$			$(\frac{g_{\pi N\Delta}}{g_{\pi NN}})^2=72/25$		

to

$$R_L(\mathbf{q},\omega)=\left(\frac{g_A}{f_\pi}\right)^2\mathbf{q}^2F^2(\mathbf{q})$$
$$\times\left[\delta(\omega-\epsilon_\mathbf{q})-\frac{1}{\pi}\frac{4}{9}\left(\frac{g_{\pi N\Delta}}{g_{\pi NN}}\right)^2\mathrm{Im}\,\frac{1}{\omega-\epsilon_{\Delta,\mathbf{q}}+i\Gamma_\Delta}\right] \qquad (15)$$

and the expression of the free-nucleon susceptibility becomes

$$\chi^N_S=-\frac{3}{16\pi^2}\frac{\langle\bar{q}q\rangle^2_{vac}}{f^4_\pi}\left(\frac{g_A}{f_\pi}\right)^2\int_0^\infty \mathrm{d}q\; q^4F^2(q)$$
$$\times\left\{\frac{3\epsilon_q^2+9\epsilon_q\omega_q+8\omega_q^2}{2\omega_q^5(\epsilon_q+\omega_q)^3}+\frac{4}{9}\left(\frac{g_{\pi N\Delta}}{g_{\pi NN}}\right)^2\int_0^\infty\mathrm{d}\omega\right.$$
$$\left.\times\left[\frac{3\omega^2+9\omega\omega_q+8\omega_q^2}{2\omega_q^5(\omega+\omega_q)^3}\right]\left(-\frac{1}{\pi}\,\mathrm{Im}\,\frac{1}{\omega-\epsilon_{\Delta,q}+i\Gamma_\Delta}\right)\right\}. \qquad (16)$$

We have checked numerically that the first method converges to this free-nucleon result in the dilute limit. Table 1 shows the results obtained. The free-nucleon susceptibility calculated from eq. (16) is displayed in the column $\rho=0$, while the other columns display the effective ones at normal nuclear-matter density. The free value depends on the parameter Λ entering in the monopole form factor $F(\mathbf{q})=\Lambda^2/(\Lambda^2+\mathbf{q}^2)$. The effective one depends in addition on the $\pi N\Delta$ coupling constant and on the Landau-Migdal parameters. We have explored the dependence on these parameters. In all cases the magnitude of the susceptibility is appreciably increased as compared to the free one, *i.e.*, the Pauli blocking is overcompensated by the effect of the increase of the pion propagator in the medium. The enhancement of the effective susceptibility is more pronounced with a harder form factor or with smaller Landau-Migdal parameters.

Figure 2 displays the evolution with density of the effective nucleon susceptibility, $\tilde{\chi}^N_S(\pi)$, for a given choice of parameters. As expected it converges to the value of eq. (16) in the $\rho\to 0$ limit. Beyond the normal density the increase with density of the effective susceptibility becomes rapid.

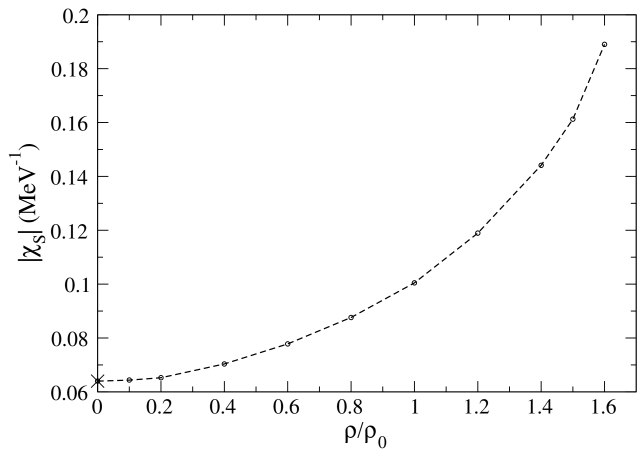

Fig. 2. Density evolution of the scalar susceptibility with the following choice of the parameters: $\Lambda=900\,\mathrm{MeV}$, $(\frac{g_{\pi N\Delta}}{g_{\pi NN}})^2=3.8$ and $g'_\Delta=0.4$. The point at $\rho=0$ is calculated from eq. (16).

The modification of the scalar susceptibility is linked to chiral symmetry restoration. In the phase of spontaneously broken symmetry the susceptibility is split in two, the "parallel" one, which is the scalar, and the "perpendicular", which is the pseudoscalar one [10]. The second one is infinite in the absence of explicit breaking, *i.e.*, in the limit $m_\pi=0$. The scalar susceptibility instead is driven by hard modes (sigma, two-pion etc.) and it is much smaller in magnitude. As the two susceptibilities merge in the restored phase a convergence between them might be a signal of a partial chiral symmetry restoration. A confirmation of this conjecture needs a study of the evolution with density of the susceptibilities up to the transition point, which is beyond the scope of this phenomenological work.

With increasing density the pseudoscalar susceptibility decreases (in magnitude) since, as was established in ref. [10], it follows the quark condensate with

$$\chi_{PS}(\rho)=\frac{\langle\bar{q}q(\rho)\rangle}{m_q}\qquad\text{and}\qquad\chi_{PS_{vac}}=\frac{\langle\bar{q}q\rangle_{vac}}{m_q}. \qquad (17)$$

At ρ_0 it has decreased by $\simeq 35\%$. On the other hand, we have seen that the presence of the nucleons which respond through their pion clouds contributes to the increase of the scalar susceptibility by (in unit of the vacuum value of the pseudoscalar susceptibility)

$$\frac{\chi_S^A}{\chi_{PS_{vac}}} = \frac{\rho_S \tilde{\chi}_S^N m_q}{\langle \overline{q}q \rangle_{vac}}. \tag{18}$$

At ρ_0 it turns out that numerically this ratio is just $\tilde{\chi}_S^N$, with $\tilde{\chi}_S^N$ expressed in MeV^{-1}, which represents a non-negligible $\simeq$ 8–10% convergence effect. We remind in this respect that there exists another and larger factor of convergence which was studied in ref. [12]. The conversion of the quark fluctuation density into nucleonic ones, *i.e.* the effect of the low-lying nuclear excitations on the QCD scalar susceptibility, produces an additional enhancement of the magnitude of the susceptibility, which at ρ_0 can be expressed in our unit as

$$\chi_S^A(\rho_0)/\chi_{PS_{vac}} \simeq 9\rho_0 \Sigma_N^2/(f_\pi^2 m_\pi^2 K), \tag{19}$$

where $K \simeq 250\,\mathrm{MeV}$ (close to the free Fermi gas value) is the incompressibility factor of nuclear matter and Σ_N the nucleon sigma commutator. It neglects the relativistic effects at ρ_0, in such a way that the nuclear responses to scalar or vector probes can be taken as identical. Moreover, this result assumes that the nucleon sigma commutator is not renormalized in the medium. The value of this ratio is $\simeq 0.56$. A more elaborate evaluation by Chanfray *et al.* [18], which takes into account in particular medium effects in the conversion coefficient, gives for the ratio $\chi_S(\rho_0)/\chi_{PS_{vac}}$ a value $\simeq 0.35$. Adding the two sources of modification and neglecting the vacuum value of χ_S which is small and depends on the uncertain sigma mass (as a magnitude order $\chi_{S_{vac}}/\chi_{PS_{vac}} \simeq 0.05$), the scalar susceptibility at ρ_0 is

$$\chi_S(\rho_0)/\chi_{PS_{vac}} \simeq 0.45, \tag{20}$$

while the pseudoscalar one in the same units is $\simeq 0.65$. The two susceptibilities which are drastically different in the vacuum become nearly equal in ordinary nuclear matter, a remarkable convergence effect. The reason for the large increase of the scalar susceptibility as compared to its vacuum value is the spectrum of scalar excitations in the nuclear medium. It encompasses nuclear states and also two–quasi-pion states which extend at lower energies than the bare two-pion states.

With increasing density the contribution of low-lying nuclear excitations to the scalar susceptibility does not increase further and even decreases, as the nuclear scalar-isoscalar response becomes collective with a repulsive residual force [19]. On the other hand, the one from the pionic excitations of the nucleons continues to increase, due to the rapid increase of the effective nucleon susceptibility. The overall effect leads a smooth behavior (up to $\rho \sim 1.6\rho_0$). For instance, with the evaluation of ref. [18] for the nuclear part, the overall susceptibility at $\rho = 1.6\rho_0$ becomes $\chi_S(1.6\rho_0)/\chi_{PS_{vac}} \simeq 0.47$, quite close to its value at ρ_0.

3 Sigma propagator and nuclear-physics implications

In nuclear physics the attraction is attributed in part to the exchange of a scalar field between nucleons. The previous description of the T-matrix in the linear sigma model incorporates a scalar sigma field, chiral partner of the pion. It is then interesting to rewrite the T-matrix in the following form which displays the propagator of the sigma field, D_σ, with the inclusion of its coupling to 2π states:

$$T(E) = \frac{6\,\lambda(E^2 - m_\pi^2)}{1 - 3\lambda G(E)} D_\sigma(E),$$
$$D_\sigma(E) = \left(E^2 - m_\sigma^2 - \frac{6\lambda^2 f_\pi^2 G(E)}{1 - 3\lambda G(E)} \right)^{-1}. \tag{21}$$

The physical interpretation of this expression is clear: the σ propagator incorporates its coupling to 2π states dressed by all rescatterings processes which are driven exclusively by the 4π contact interaction. At $E = 0$ we have

$$-D_\sigma(0) = \frac{1}{m_\sigma^2 + 3\lambda(m_\sigma^2 - m_\pi^2)\frac{G}{1-3\lambda G}} = \frac{1 - 3\lambda G}{m_\sigma^2 - 3\lambda m_\pi^2 G}$$
$$\simeq \frac{1}{m_\sigma^2} - \frac{3\lambda G}{m_\sigma^2}. \tag{22}$$

The medium correction to D_σ from the coupling of the σ to 2π states is

$$\Delta D_\sigma(0) = \frac{3\Delta G(0)}{2f_\pi^2}, \tag{23}$$

which represents at ρ_0 a correction of $\simeq 4 \cdot 10^{-6}\,\mathrm{MeV}^{-2}$. For comparison, the bare vacuum value with a sigma mass of 700 MeV is $\simeq 2 \cdot 10^{-6}\,\mathrm{MeV}^{-2}$, smaller than the medium contribution.

Notice that without the $\pi\pi$ rescattering correction the numerical value of ΔG at ρ_0 is such that the denominator in the expression of D_σ (eq. (21)) would be negative. The existence of a singularity in the σ propagator implies an instability with respect to a 2π isoscalar soft mode, which was discussed by Aouissat *et al.* [20]. These authors have argued that the pion rescattering effect (due to the 4π contact term) in the sub-threshold region could eliminate the instability, as is the case in our expression (21) at $E = 0$.

We have seen above that the large polarization of the nucleon through the pion cloud has a large effect on the σ propagation. The following question naturally arises: is the large medium modification of the σ propagator reflected in the NN interaction? At first sight it is natural to believe that the scalar NN potential is affected in the same way as the σ propagator, which would lead to strong many-body forces. The answer to the question is closely related to the problem of the identity between the scalar meson responsible for the nuclear binding and the sigma, chiral partner of the pion. The pure identity between the scalar meson which contributes to the nuclear binding and the chiral partner of the pion is excluded by chiral constraints, emphasized by Birse [21]. It would lead to the presence of a term of order m_π in the NN interaction, which is not allowed. Nevertheless, it is possible to describe the NN attraction in the linear sigma model as showed by Chanfray *et al.* [12]. By going from Cartesian to polar coordinates, these authors introduced a new scalar field called $S = f_\pi + s$. This field is associated with the radius of the chiral circle and it is a chiral invariant while the σ field is not. They suggested that the scalar meson of nuclear physics should be identified with this new scalar field. The justification will be given later. More precisely, the nuclear attraction arises from the mean (negative) value $\bar{s}$, and the effective nucleon mass is $M_N^* = M_N + g_S\bar{s}$. Actually, this new formulation transforms the original linear realization of chiral symmetry into a non-linear one. Consequently, chiral constraints are automatically respected, as those mentioned by Birse. Moreover, the coupling constant of the s field to the nucleon, $g_S = M_N/f_\pi \simeq 10$, is not incompatible with the phenomenology of quantum hadrodynamics [22].

The passage to polar coordinates cannot affect the physics. For instance, the T-matrix for on-shell pions must be independent of the representation. It is therefore interesting to rewrite it in a form which displays the propagator of the s field, the relevant quantity for nuclear physics. For this purpose we now express the Lagrangian of the linear sigma model in terms of the polar coordinates, *i.e.*, the s field and the new pion field ϕ which is directly related to the chiral angle, being

$$\sigma = (f_\pi + s)\cos\left(\frac{\phi}{f_\pi}\right), \qquad \vec{\pi} = (f_\pi + s)\hat{\phi}\sin\left(\frac{\phi}{f_\pi}\right). \tag{24}$$

This Lagrangian has been given in [12] and we restrict ourselves to pieces relevant for our purpose. We first note that the bare masses for the s and σ fields are the same. For the 4π contact term we recover the standard non-linear sigma model result with contain derivative terms. In addition, we get a $s\pi\pi$ coupling piece of the derivative type which contains

$$\mathcal{L} = \frac{s}{f_\pi}\left(\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi} - \frac{1}{2}m_\pi^2\phi^2\right). \tag{25}$$

As an illustration, summing contact and s exchange pieces the Born amplitude reads

$$V(E) = \frac{6}{f_\pi^2}\left(-(E^2 - m_\pi^2) + \frac{(E^2 - m_\pi^2)^2}{E^2 - m_\sigma^2}\right) \tag{26}$$

which reproduces the previous result of eq. (1), as expected; only the decomposition has changed. The same holds for the T-matrix. The interest lies in the identification of the propagator D_s of the chiral invariant scalar field s. We rewrite the T-matrix in a form which displays the coupling, $E^2 - m_\pi^2$, of the s field to two pions and the $\pi\pi$ rescattering through the new contact interaction (first

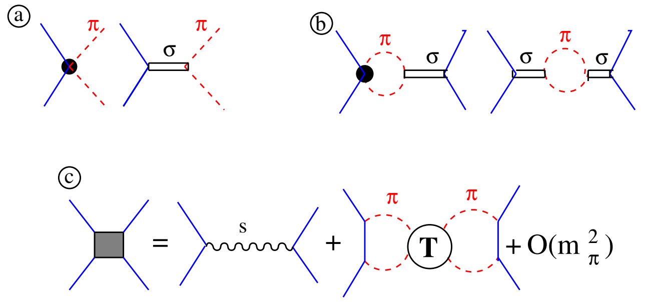

Fig. 3. a) Compensating contributions to the πN amplitude with pseudo-scalar coupling; the blob represents the $N\bar{N}$ intermediate state. b) Corresponding compensation in the NN interaction leading to the suppression of the 2π dressing of the σ propagator. c) Resulting NN potential with undressed σ exchange, *i.e.* s exchange, and correlated two-pion exchange with in-medium modified $\pi\pi$ T-matrix (here the intermediate states are nucleon or Δ ones).

term of eq. (26)). This new decomposition reads

$$T(E) = \frac{6\,\lambda(E^2 - m_\pi^2)}{1 + \frac{3}{2f_\pi^2}(E^2 - m_\pi^2)\,G(E)}D_s(E)$$
$$D_s(E) = \left(E^2 - m_\sigma^2 - \frac{3}{2f_\pi^2}(E^2 - m_\pi^2)^2 \times\frac{G(E)}{1 + \frac{3}{2}\frac{E^2 - m_\pi^2}{f_\pi^2}G(E)}\right)^{-1}. \tag{27}$$

We consider the zero-energy case, $E = 0$, and compare the propagators of the s and σ fields. They differ in an essential way with respect to their coupling to two pions. For the s, the coupling vanishes in the chiral limit, hence it is small and it can be ignored, while it is large for the σ. This difference is due to the chiral invariant character of the s field. As the NN scalar potential is linked to the exchange of the s mode, it is not modified by the medium effects discussed previously, which affect the σ propagation through its coupling to 2π states.

We now come to the justification of the identification of the nuclear attraction with the s field exchange. The physics cannot depend on field transformation from Cartesian to polar coordinates. Hence, the same conclusion about the stability of the NN potential should be reached also in the original linear formulation. In this case the nucleons exchange a σ with its $\pi\pi$ dressing but the consistency of the model also implies other exchanges with resulting delicate compensations [23]. Their origin is the well-known pair suppression, in the case of pseudo-scalar coupling, by σ exchange for the πN amplitude. As depicted in fig. 3b, this translates into the elimination of the sigma dressing in the NN interaction. We have explicitly checked that this cancellation holds to all orders in the dressing of the sigma. The net result amounts to the exchange of the s mode and hence to the identification of Chanfray *et al.* [12]. Their formulation provides a very economical way to incorporate all the cancellations inherent to the linear realization, and hence the requirements of chiral symmetry. In addition to s exchange it is clear that the standard correlated two-pion exchange with pseudo-vector πNN coupling remains (see fig. 3c). It undergoes the medium modifications of the $\pi\pi$ T-matrix discussed in sect. 2. This effect has been worked out in [24]. The overall change of the NN potential depends very much on the relative weight of the two components, s exchange and correlated 2π exchange, *i.e.*, on the sigma mass.

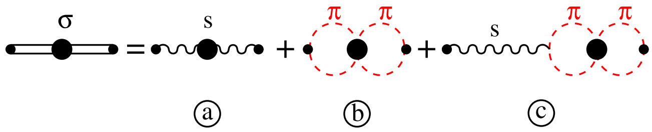

Fig. 4. Decomposition of the dressed σ propagator into: a) the dressed s propagator, b) the dressed two-pion propagator and c) the mixed term with a bare s propagator and a dressed two-pion one.

4 Two components description of the σ propagator and conclusion

In this last section we perform the separation of the σ propagator into two components which are weakly coupled to each other. The decomposition reads

$$D_\sigma(E) = D_s(E) + \frac{3}{2f_\pi^2}\left(1 - 2\frac{E^2 - m_\pi^2}{E^2 - m_\sigma^2}\right)\tilde{G}(E), \quad (28)$$

where $\tilde{G}$ is the fully dressed 2π propagator which obeys the equation

$$\tilde{G} = G + \frac{1}{2}\, G\, V\, \tilde{G}, \quad (29)$$

where V has been given in eq. (1). The physical interpretation of the relation (29) is simple and it is illustrated in fig. 4. The sigma propagator contains three parts: first, the dressed s propagator derivatively coupled to two-pion states (fig. 4a), second, the fully dressed two-pion propagator (fig. 4b) and third, a mixed term with a bare s propagator $D_s^0 = (E^2 - m_\sigma^2)^{-1}$ coupled to a fully dressed two-pion states (fig. 4c). The two-pion propagator is the only piece which survives when m_σ becomes infinite. To order m_π^2, the medium effects that we have introduced in this work appear only in the two-pion component. In the NN interaction, we have shown that this two-pion component (taken at $E = 0$) does not enter and only the first one is active and to order m_π^2 is not renormalized.

In summary, the unifying theme of the present work has been the question of the two-pion propagator in the nuclear medium. It is modified by the dressing of the pionic lines by particle-hole bubbles. The 2π propagator appears in many problems. One of them, already explored, is related to the 2π production experiments on nuclei. It enters through its influence on the $\pi\pi$ T-matrix and its nuclear modification is responsible for the softening of the scalar strength observed in these data [25–27].

In the present work we have focused on the case of zero four-momentum for the two-pion propagator since this kinematical situation corresponds to the problem of the QCD scalar susceptibility. Here the dressing of the pion lines by $p-h$ bubbles introduces in the nuclear susceptibility the response of the individual nucleons to a change of the light quark mass. This response is dominated by the nucleon pion cloud. As the cloud is polarized in the medium we have shown that this response undergoes a large renormalization in the sense of an enhancement of its magnitude, which can reach typically $\simeq 50\%$ at $\rho = \rho_0$. The contribution of the nucleon scalar susceptibility brings the nuclear one closer to the pseudoscalar one. Thus, the nuclear pions have a double role in the restoration effects: first, they participate in the decrease of the quark condensate, *i.e.* also of the pseudoscalar susceptibility which follows the condensate. In addition through the softening of the scalar strength they increase the scalar susceptibility, thus participate further in the convergence between the two susceptibilities, which then become nearly equal at the ordinary density, a remarkable convergence effect which signals chiral symmetry restoration.

There is indeed a message about chiral symmetry restoration contained in the 2π production experiments on nuclei. A softening of strength, as is observed, naturally translates into an increase (in magnitude) of the corresponding scalar susceptibility, which is the inverse energy weighted sum rule.

Another quantity to be influenced by the 2π propagator is the propagator of the sigma meson, chiral partner of the pion, which is coupled to two-pion states. We have shown that, at zero four-momentum, the change of the 2π propagator in the medium produces a major modification of the σ propagator which can triple its magnitude.

Our next step has been the investigation of the consequences for the NN interaction in the nuclear medium. The scalar meson responsible for the nuclear attraction cannot be the previous sigma meson, but it has to be a chiral invariant scalar field. We have shown in a previous work that such an object can be found in the linear sigma model, being related to the fluctuation of the radius of the chiral circle. Contrary to the σ this invariant field is weakly coupled to the pion. Therefore, it does not undergo the large medium modification of the σ, insuring the stability of the corresponding s exhange of the NN interaction in the nuclear medium. In a forthcoming work we plan to incorporate also the pion loops on top of our hadrodynamic mean-field description of the nuclear binding [10]. This will allow the systematic inclusion of a correlated two-pion term with its in-medium modification.

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